PRECALCULUS I/MATH 126

SHANNON MYERS

- π 100 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π YOU MAY USE A SCIENTIFIC AND/OR A TI-83/84/85/86 CALCULATOR
- π PROVIDE EXACT ANSWERS UNLESS OTHERWISE INDICATED

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED…THIS MEANS NO BATHROOM BREAKS!

NAME Key

PRECALCULUS/MATH 126/MYERS NAME Key

PLEASE MAKE SURE YOU ARE TAKING THE CORRECT EXAM!!!

EXAM 1/100 POINTS POSSIBLE

CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANIZED, EASY TO READ FASHION.

1. (9 POINTS) Let $g(x) = x^3 + x$.

   a. (3 POINTS) Is g odd, even, or neither odd nor even. Please explain.

1) Test for even: Does $g(-x) = g(x)$?

$g(-x) = (-x)^3 + (-x)$

$g(-x) = -x^3 - x$

Since $g(x) = x^3 + x$, g is NOT even.

2) Test for odd: Is $g(-x) = -g(x)$?

$-g(x) = -[x^3 + x]$

$-g(x) = -x^3 - x$

We know from the test for if a function is even that $g(-x) = -x^3 - x$, so

g is odd.

   b. (3 POINTS) Find the average rate of change from -4 to 1.

Average rate of change is equivalent to the slope of the line secant to g.

$m_{sec} = \frac{g(1) - g(-4)}{(1) - (-4)}$

$m_{sec} = \frac{2 - (-68)}{1 + 4}$

$m_{sec} = \frac{70}{5}$

$m_{sec} = 14$

scratch work

$g(1) = (1)^3 + (1) = 2$

$g(-4) = (-4)^3 + (-4) = -68$

The average rate of change from -4 to 1 is 14.

   c. (3 POINTS) Find an equation of the secant line containing $(-4, g(-4))$ and $(1, g(1))$. Give your result in the **point-slope form of the line**.

$m = 14$ $(-4, -68)$ $(1, 2)$

$y - y_1 = m(x - x_1)$

For $(-4, -68)$:

$y - (-68) = 14(x - (-4))$

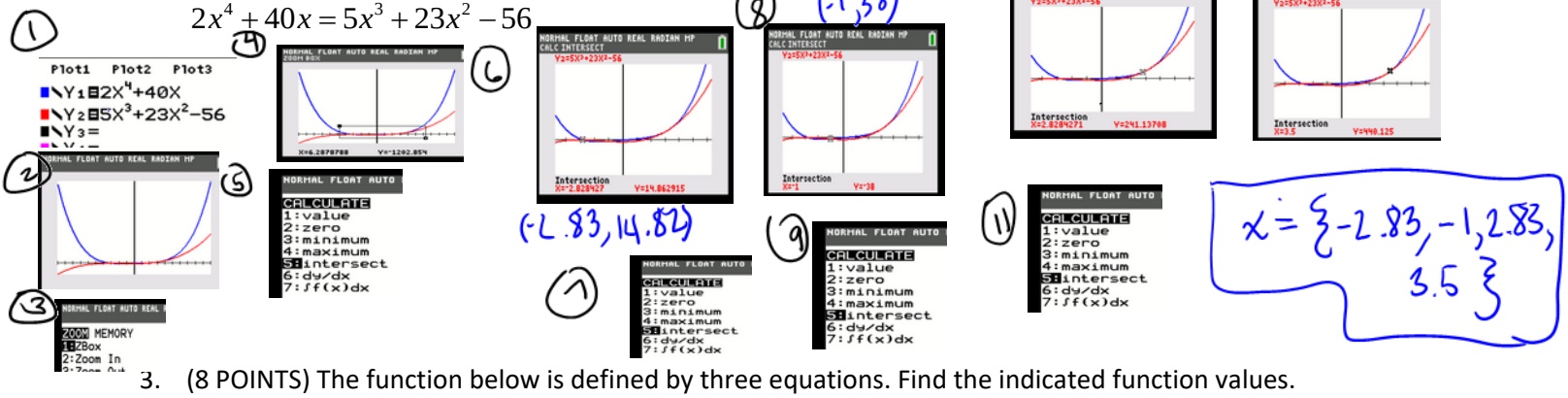
$y + 68 = 14(x + 4)$

For $(1, 2)$:

$y - 2 = 14(x - 1)$

2. (4 POINTS) Use a graphing calculator to approximate the real solutions, if any, of the given equation rounded to **two decimal places**. All solutions lie between -10 and 10.

$$2x^4 + 40x = 5x^3 + 23x^2 - 56$$



$x = \{-2.83, -1, 2.83, 3.5\}$

3. (8 POINTS) The function below is defined by three equations. Find the indicated function values.

$$f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 2 \\ 8 & \text{if } x > 2 \end{cases}$$

$\sqrt[3]{-8} = -2$

$(-1)^2 = 1$, $(0)^2 = 0$

a. $f(-1) =$ 1

b. $f(-8) =$ -2

c. $f(0) =$ 0

d. $f(-8) + f(4) =$ 6

$(-2) + 8$

4. (8 POINTS) Find the difference quotient of f; that is, find $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$, for the following function. Simplify your answer.

$f(x) = 3 - x^2$

Scratch work:

$f(x+h) = 3 - (x+h)^2$

$= 3 - (x^2 + 2xh + h^2)$

$= 3 - x^2 - 2xh - h^2$

$f(x) = 3 - x^2$

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{(3 - x^2 - 2xh - h^2) - (3 - x^2)}{h}$$

$$= \frac{3 - x^2 - 2xh - h^2 - 3 + x^2}{h}$$

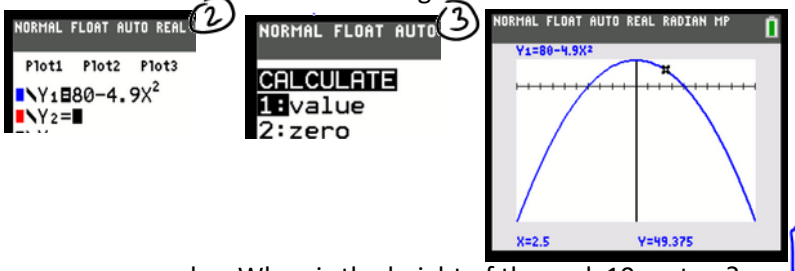
$$= \frac{-2xh - h^2}{h}$$

$$= \frac{-h(2x+h)}{h}$$

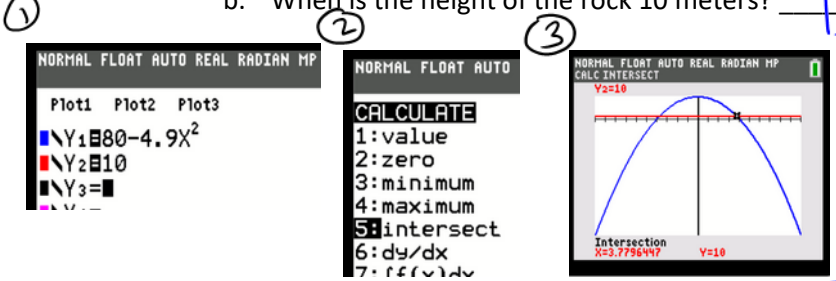
$$= -2x - h$$

5. (9 POINTS) If a rock falls from a height of 80 meters on Earth, the height H in meters after *x* seconds is approximately $H(x) = 80 - 4.9x^2$. Round your answers to **three decimal places**. Give the appropriate **units** with your answers.

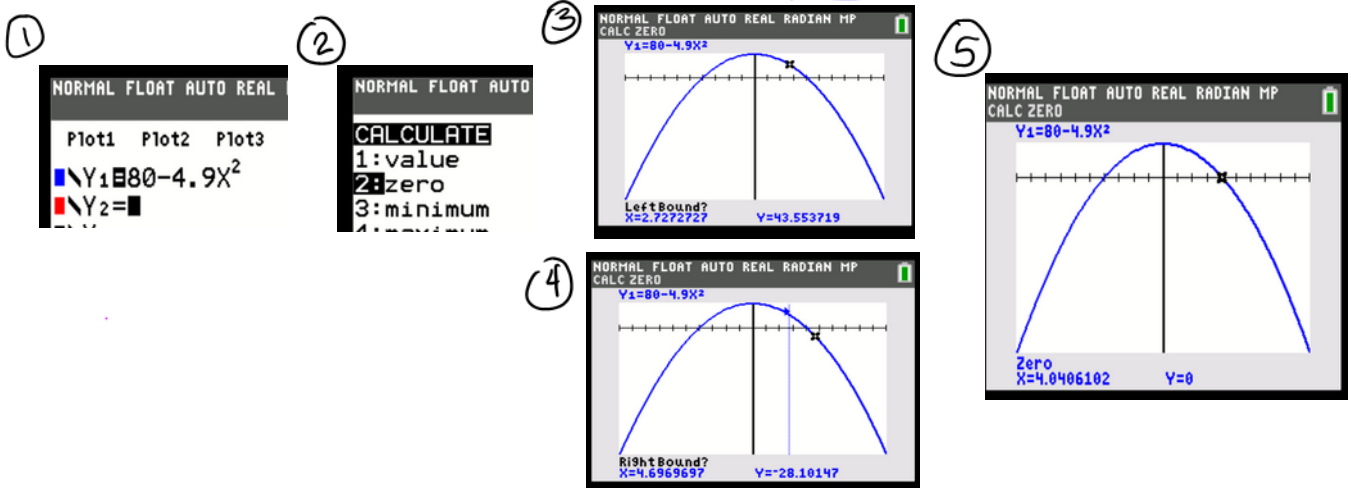
a. What is the height of the rock when $x = 2.5$ seconds? 49.375 meters



b. When is the height of the rock 10 meters? 3.780 seconds

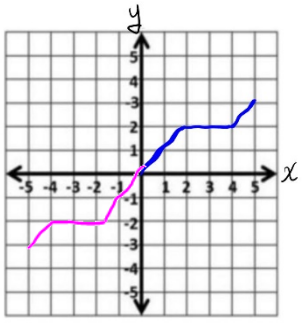


c. When does the rock hit the ground? 4.041 seconds

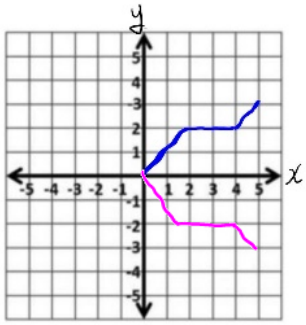


6. (6 POINTS) Complete the graph so that the graph is symmetric with respect to the:

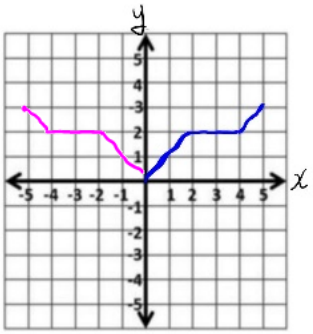
a. Origin



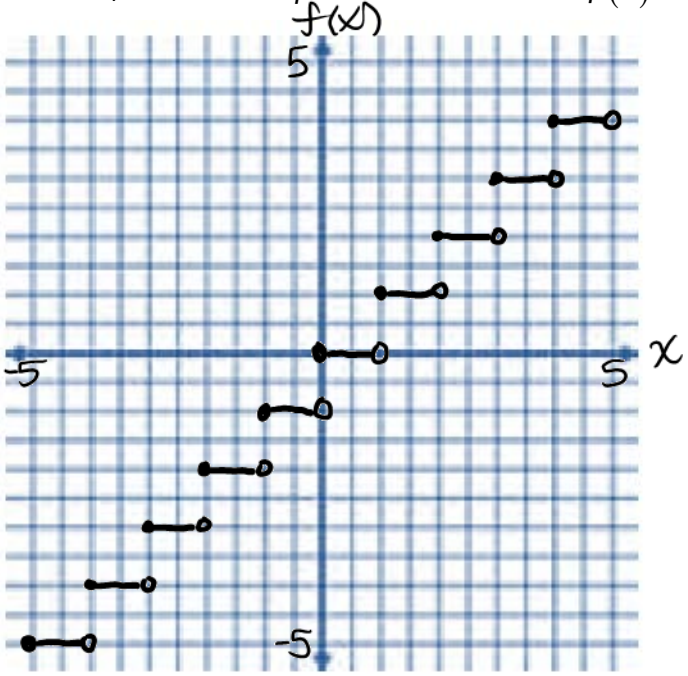
b. *x*-axis



c. *y*-axis



7. (8 POINTS) The function f is defined as follows: $f(x)=\text{int}(x)$ (also notated $f(x)=[\![x]\!]$).



a. (4 POINTS) Graph the function. Be sure to label axes and scale.

b. (2 POINTS)What is the domain? $\mathbb{R}$ or $(-\infty, \infty)$

c. (2 POINTS) Is f continuous on its domain? no

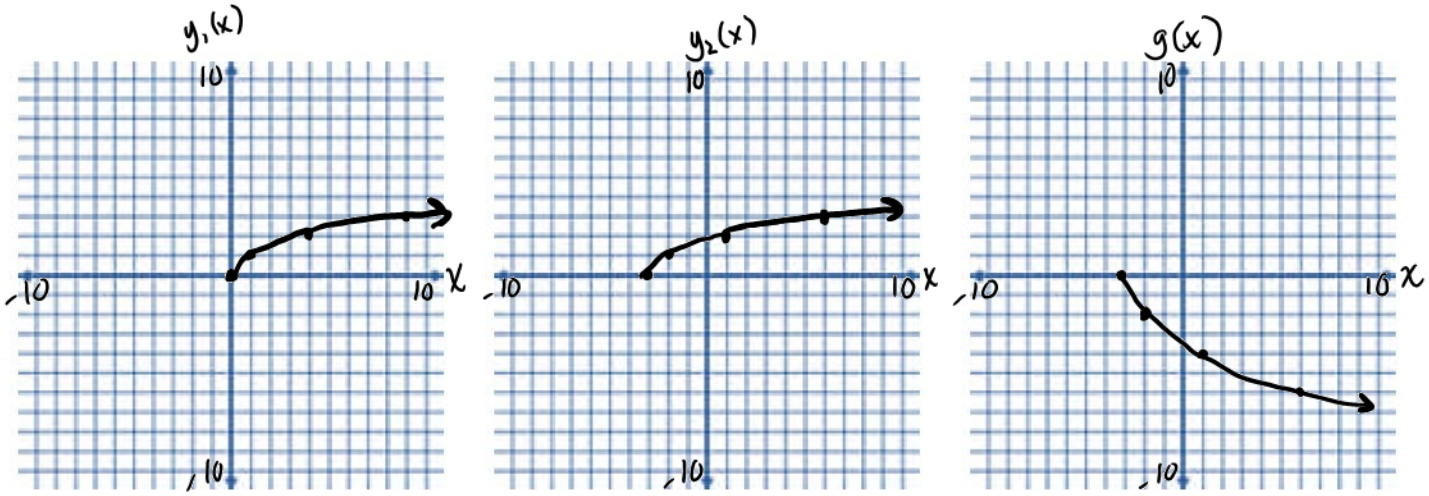
8. (4 POINTS) Give the domain of $f(x)=\dfrac{x}{x+3}$ in interval notation.

$x+3 \neq 0$

$x \neq -3$

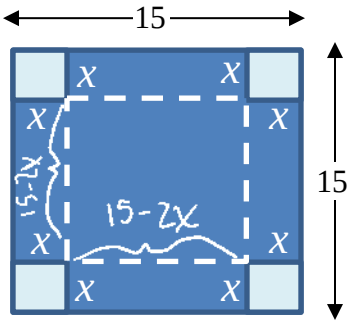
$(-\infty, -3) \cup (-3, \infty)$

9. (9 POINTS) Graph $g(x) = -2\sqrt{x+3}$ **by hand, using transformations**. Fill in the blanks below to indicate the first two graphs. DO NOT USE YOUR GRAPHING CALCULATOR!



$y_1 =$ $\sqrt{x}$ $y_2 =$ $\sqrt{x+3}$ $g(x) = -2\sqrt{x+3}$

10. (8 POINTS) An open box with a square base is to be made from a piece of cardboard 15 inches on a side by cutting out a square from each corner and turning up the sides.



a. (4 POINTS) Express the volume V of the box as a function of the length x.

$V(x) = x(15-2x)^2$

or

$V(x) = 225x - 60x^2 + 4x^3$

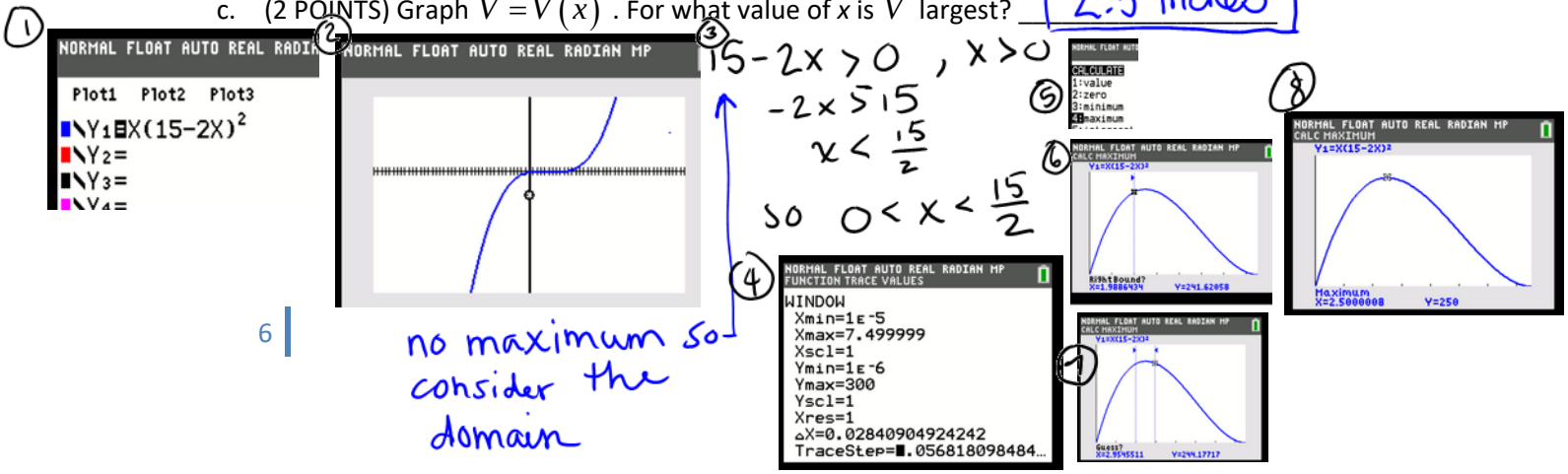
b. (2 POINTS) What is the volume if a 5-inch square is cut out?

$V(5) = 5[15-2(5)]^2$

$V(5) = 5(15-10)^2$

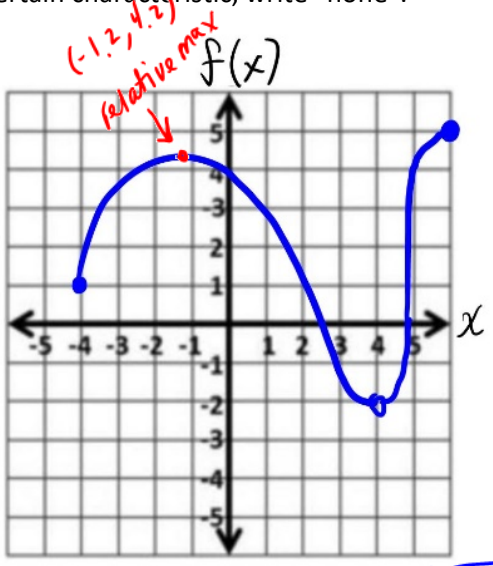
$V(5) = 125 \text{ in}^3$

c. (2 POINTS) Graph $V = V(x)$. For what value of x is V largest? 2.5 inches



no maximum so consider the domain

11. (9 POINTS) Consider the graph of $f(x)$ below. Round your answer to the nearest tenth. If the graph does not have a certain characteristic, write "none".



a. What are the zeros of f? $x = 2.5, 4.8$

b. $f(0) =$ 4

c. What is the absolute maximum? 5

d. What is the absolute minimum? NONE

e. On what interval(s) is f decreasing? $(-1.2, 4)$

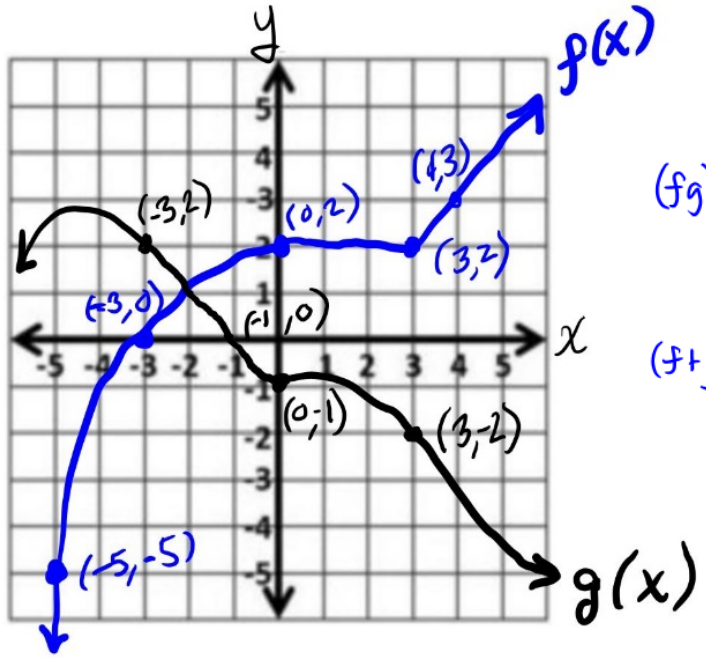
f. On what interval(s) is f increasing? $(-4, -1.2) \cup (4, 6)$

g. What is the domain of f? $[-4, 4) \cup (4, 6]$ or $\{x : x \text{ is a real number}, x \neq 4\}$

h. What is the range of f? $(-2, 5]$ or $\{y : y \text{ is a real number}, -2 < y \leq 5\}$

i. For what values of x is $f(x) < 0$? $(2.5, 4) \cup (4, 4.8)$

12. (6 POINTS) Use the graph of the functions to answer the following questions.



a. $(fg)(3) =$ -4

$(fg)(3) = f(3)g(3)$
$= (2)(-2)$
$= -4$

b. $(f+g)(0) =$ 1

$(f+g)(0) = f(0) + g(0)$
$= 2 + (-1)$
$= 1$

c. $\left(\frac{f}{g}\right)(-3) =$ 0

$\left(\frac{f}{g}\right)(-3) = \frac{f(-3)}{g(-3)}$
$= \frac{0}{2}$
$= 0$

13. (6 POINTS) Find the equation of the line that is parallel to the line $y = -\frac{5}{2}x - 1$ and passes through the point $(-1, 4)$.

1) parallel lines have the same slope, so the slope of this line is $-\frac{5}{2}$, which is the slope of $y = -\frac{5}{2}x - 1$.

2) $y - y_1 = m(x - x_1)$

$y - 4 = -\frac{5}{2}(x - (-1))$

$y - 4 = -\frac{5}{2}(x + 1)$

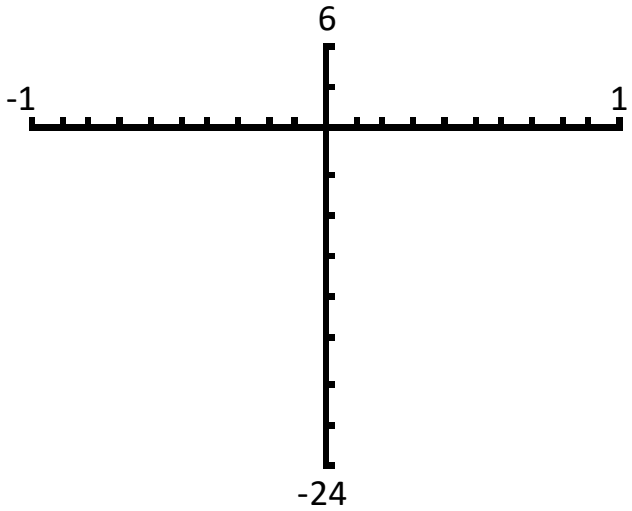
or

$y = -\frac{5}{2}x + \frac{3}{2}$

or

$5x + 2y = \frac{3}{2}$

14. (6 POINTS) Determine the viewing window used.



a. Xmin = -1

b. Xmax = 1

c. Xscl = $\frac{1}{10}$

d. Ymin = -24

e. Ymax = 6

f. Yscl = 3